Model Order Reduction and Compressive Sampling

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This abstract presents practical numerical recipes for simulating high-temperature and nonequilibrium quantum spin systems that are continuously measured and controlled. The notion of a "spin system" is broadly conceived, in order to encompass macroscopic test masses as the limiting case of large-j spins. The simulation technique has three stages: first the deliberate introduction of noise into the simulation, then the conversion of that noise into an informatically equivalent continuous measurement and control process, and finally, projection of the trajectory onto a Kahlerian state-space manifold having reduced dimensionality and possessing a Kahler potential of multilinear (i.e., product-sum) functional form. These state-spaces can be regarded as ruled algebraic varieties upon which a projective quantum model order reduction (QMOR) is performed. The Riemannian sectional curvature of ruled Kahlerian varieties is analyzed, and proved to be non-positive upon all sections that contain a rule. It is further shown that the class of ruled Kahlerian state-spaces includes the Slater determinant wave-functions of quantum chemistry as a special case, and that these Slater determinant manifolds have a Fubini-Study metric that is Kahler-Einstein; hence they are solitons under Ricci flow. It is suggested that these negative sectional curvature properties geometrically account for the fidelity, efficiency, and robustness of projective trajectory simulation on ruled Kahlerian state-spaces. Some implications of trajectory compression for geometric quantum mechanics are discussed. The resulting simulation formalism is used to construct a positive P-representation for the thermal density matrix and to derive a quantum limit for force noise and measurement noise in monitoring both macroscopic and microscopic test-masses; this quantum noise limit is shown to be consistent with well-established quantum noise limits for linear ampliers and for monitoring linear dynamical systems. Single-spin detection by magnetic resonance force microscopy (MRFM) is then simulated, and the data statistics are shown to be those of a random telegraph signal with additive white noise, to all orders, in excellent agreement with experimental results. Then a larger-scale spin-dust model is simulated, having no spatial symmetry and no spatial ordering; the high-fidelity projection of numerically computed quantum trajectories onto low-dimensionality Kahler state-space manifolds is demonstrated. Finally, the high-fidelity reconstruction of quantum trajectories from sparse random projections is demonstrated, the onset of Donoho-Stodden breakdown at the Candes-Tao sparsity limit is observed, and methods for quantum state optimization by Dantzig selection are given.